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GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN

(Autonomous)

(Affiliated to Andhra University, Visakhapatnam)

II B.Tech. - I Semester Regular Examinations, Nov – 2025

COMPLEX VARIABLES AND STATISTICAL METHODS

(EEE Branch)

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. Max Marks: 70

UNIT-I

1. a. Construct the analytic function f(z) whose real part is $e^x(xcosy - ysiny)$ (7M)

   b. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at that point. (7M)

OR

2. a. Evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is $|z - i| = 2$ (7M)

   b. Evaluate using Cauchy Integral formula, $\int_C \frac{cos\pi z^2}{(z-1)(z-2)} dz$ where $C: |z| = 4$ (7M)

UNIT-II

3. a. Using Cauchy Residue Theorem, evaluate $\oint_C \frac{12z-7}{(2z+3)(z-1)^2} dz$ where $C: x^2 + y^2 = 4$ (7M)

   b. Evaluate $\int_0^\infty \frac{1}{(x^2+a^2)^2} dx$ (7M)

OR

4. a. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the region $2 < |z| < 3$ (7M)

   b. Find the sum of residues of $f(z) = \frac{z^3}{(z-2)^2(z-3)}$ at its poles. (7M)

UNIT-III

5. a. In a certain college, 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body. What is the probability that mathematics is being studied? If a student is selected at random and is found to be studying mathematics then find the probability that the student is a girl. (7M)

   b. A random variable X is defined as sum of the numbers on the faces when two dice are thrown. Find the mean of X. (7M)

OR

6. a. The probability of a man hitting a target is 1/3. (i) If he fires 5 times, what is the probability of his hitting the target at least twice? (ii) How many times must he fire so that the probability of his hitting the target at least once is more than 90%? (7M)

   b. If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3kgs, how many students have masses.

   i) Greater than 72 kg

   ii) Less than or equal to 64 kg

   (iii) Between 65 and 71 kg inclusive. (7M)

UNIT-IV

7. a. A population consists of five numbers 2, 3, 6, 8 and 11. Consider all possible samples of size two which can be drawn without replacement from this population. Find.
   - (i) The mean of the population.
   - (ii) The Standard deviation of the population.
   - (iii) The mean of the sampling distribution of means and
   - (iv) The standard deviation of the sampling distribution of means. **(7M)**

   b. Determine the probability that the sample mean area covered by sample of 40 one litre paint boxes will be between 510 to 520 square feet given that a one litre of such paint box covers on the average 513.3 square feet with s.d. of 31.5 s.ft. **(7M)**

OR

8. a. A random sample of size 100 is taken from a population with $\sigma = 5.1$. Given that the sample mean is $\bar{x} = 21.6$. Construct a 95 % confidence interval for the population mean μ. **(7M)**

   b. A random sample of size 100 has a standard deviation of 5. What can you say about the maximum error with 95 % confidence. **(7M)**

UNIT-V

9. a. A simple sample of heights of 6400 Englishmen has a mean of 67.85 inches and a mean of 67.85 inches and a S.D. of 2.56 inches while a simple sample of heights of 1600 Australian has a mean of 68.55 inches and a S.D. of 2.52 inches. Do the data indicate that the Australians are on the average taller than the Englishmen at 5% level of significance? **(7M)**

   b. The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? **(7M)**

OR

10. a. In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers? **(7M)**

    b. The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines:

Mine 1	8260	8130	8350	8070	8340	-----
Mine2	7950	7890	7900	8140	7920	7840

Use 0.02 LOS to test whether it is reasonable to assume that the variances of the two population samples are equal. **(7M)**

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COMPLEX VARIABLES AND STATISTICAL METHODS

(EEE branch)

1. All questions carry equal marks
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Time: 3 Hrs Max. Marks: 70

UNIT-I

1. a. Construct the analytic function $f(z)$ whose real part is $e^x(x\cos y - y\sin y)$.

Solution: Given $u(x,y) = e^x(x\cos y - y\sin y)$.

$$\begin{aligned}\frac{\partial u}{\partial x} &= e^x(\cos y) + e^x(x\cos y - y\sin y)\\ &= e^x\big[(1+x)\cos y - y\sin y\big]\\ \frac{\partial u}{\partial y} &= e^x\big[-x\sin y - y\cos y - \sin y\big].\end{aligned}$$

Let v be the harmonic conjugate of u. Then

$$\begin{aligned}f'(z) &= \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\\ &= \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} \quad \text{(Using C-R equations)}\\ &= e^x\big[(1+x)\cos y - y\sin y\big] - ie^x\big[-x\sin y - y\cos y - \sin y\big]\\ &= e^x\big[(1+x)\cos y - y\sin y\big] + ie^x\big[x\sin y + y\cos y + \sin y\big].\end{aligned}$$

By Milne-Thomson's method, $f'(z)$ is expressed in terms of z by replacing x by z and y by 0. Hence

$$f'(z) = e^z\big[(1+z) - 0\big] + ie^z\big[0+0+0\big] = e^z(1+z).$$

Integrating w.r.t. z, we get

$$\begin{aligned}f(z) &= \int e^z(1+z)dz + c \qquad \left[\because \int e^{ax}f(x)dx = e^{ax}\left[\frac{f(x)}{a} - \frac{f'(x)}{a^2} + \cdots\right]\right]\\ &= ze^z + c\end{aligned}$$

1. b. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at that point.

Solution: Let $f(z) = u(x,y) + iv(x,y) = \sqrt{|xy|}$ so that $u(x,y) = \sqrt{|xy|}$ and $v(x,y) = 0$.

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{u(x+\Delta x, y) - u(x,y)}{\Delta x}$$

At the origin, $$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{u(\Delta x, 0) - u(0,0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{0-0}{\Delta x} = 0$$

$$\frac{\partial u}{\partial y} = \lim_{\Delta y \to 0} \frac{u(x, y+\Delta y) - u(x,y)}{\Delta y}$$

At the origin, $$\frac{\partial u}{\partial y} = \lim_{\Delta y \to 0} \frac{u(0, \Delta y) - u(0,0)}{\Delta y} = \lim_{\Delta y \to 0} \frac{0-0}{\Delta y} = 0$$

At the origin, $$\frac{\partial v}{\partial x} = 0 \quad [\because v = 0]$$

At the origin, $$\frac{\partial v}{\partial y} = 0.$$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Hence, the Cauchy-Riemann equations are satisfied at the origin.

Now,

$$f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z - 0} = \lim_{z \to 0} \frac{\sqrt{|xy|} - 0}{x + iy} = \lim_{z \to 0} \frac{\sqrt{|xy|}}{x + iy}.$$

If $z \to 0$ along the straight line $y = mx$, then $x \to 0, y \to 0$.

$$\therefore f'(0) = \lim_{x \to 0} \frac{\sqrt{|mx^2|}}{x + imx} = \lim_{x \to 0} \frac{x\sqrt{|m|}}{x(1+im)} = \lim_{x \to 0} \frac{\sqrt{|m|}}{1+im} = \frac{\sqrt{|m|}}{1+im}.$$

Since the limit depends on m, for different paths, we have different limits. So the limit is not unique and hence $f'(0)$ does not exist. Thus $f(z)$ is not analytic at the origin, even though the C-R equations are satisfied at the origin.

2. a. Evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)} dz$, where C is $|z - i| = 2$.

Solution: Let $I = \oint_C \frac{z-1}{(z+1)^2(z-2)} dz$.

C is the circle $|z - i| = 2$ with centre at $(0,1)$ and a radius of 2.

The singular points are given by $(z+1)^2(z-2) = 0 \Rightarrow z = -1, 2$.

If $z = 1$, then $|z - i| = |-1 - i| = \sqrt{2} < 2$.

$\therefore z = -1$ lies inside C.

If $z = 2$, then $|z - i| = |2 - i| = \sqrt{5} > 2$.

$\therefore z = 2$ lies outside C.

Let $f(z) = \dfrac{z-1}{z-2}$. Therefore, $f(z)$ is analytic inside and on C.

Now $f(z) = \dfrac{z-1}{z-2} \Rightarrow f'(z) = \dfrac{(z-2)\cdot 1 - (z-1)\cdot 1}{(z-2)^2} = -\dfrac{1}{(z-2)^2}$

and $f'(-1) = -\dfrac{1}{(-1-2)^2} = -\dfrac{1}{9}$.

$\therefore$ By Cauchy's integral formula, we have

$$\oint_C \frac{f(z)}{(z-a)^{n+1}}dz = \frac{2\pi i}{n!}f^{(n)}(a)$$

$$\Rightarrow \oint_C \frac{f(z)}{(z+1)^2}dz = \frac{2\pi i}{1!}f'(-1) = 2\pi i f'(-1)$$

$$\therefore \oint_C \frac{z-1}{(z+1)^2(z-2)}dz = 2\pi i\left(\frac{-1}{9}\right) = -\frac{2\pi i}{9}.$$

2. b. Evaluate using Cauchy integral formula, $\oint_C \dfrac{\cos \pi z^2}{(z-1)(z-2)}dz$, where $C : |z| = 4$.

Solution: Let $I = \oint_C \dfrac{\cos \pi z^2}{(z-1)(z-2)}dz$

The integrand has singularities at $z = 1$ and $z = 2$.

C is the circle $|z| = 4$ with centre at $(0, 0)$ and a radius of 3.

For $z = 1$, $|z| = |1| = 1 < 4$. Hence, $z = 1$ lies inside C.

For $z = 2$, $|z| = |2| = 2 < 4$. Hence, $z = 2$ lies inside C.

Both are lies inside $|z| = 4$.

Let $f(z) = \cos \pi z^2$.

$\therefore f(z)$ is analytic inside and on C.

$$\frac{1}{(z-1)(z-2)} = \frac{(z-1)-(z-2)}{(z-1)(z-2)} = \frac{1}{z-2} - \frac{1}{z-1}$$

By Cauchy's integral formula,

$$\oint_C \frac{f(z)}{z-a}dz = 2\pi i f(a)$$

$$\Rightarrow \oint_C \frac{f(z)}{(z-1)(z-2)}dz = \oint_C \frac{f(z)}{z-2}dz - \oint_C \frac{f(z)}{z-1}dz$$

$$\Rightarrow \oint_C \frac{\cos \pi z^2}{(z-1)(z-2)}dz = 2\pi i f(2) - 2\pi i f(1)$$

$$= 2\pi i\big[\cos 4\pi - \cos \pi\big]$$

$$= 2\pi i\big[1-(0-1)\big]$$

$$= 4\pi i.$$

UNIT-II

3. a. Using Cauchy Residue theorem, evaluate $\oint_C \frac{12z-7}{(2z+3)(z-1)^2}dz$, where C : $x^2+y^2=4$.

Solution: Let $f(z) = \frac{12z-7}{(2z+3)(z-1)^2}$.

The poles of $f(z)$ are given by $(2z+3)(z-1)^2 = 0 \Rightarrow z = 1, z = -\frac{3}{2}$.

C is the circle $|z| = 2$ with centre at $(0,0)$ and a radius of 2.

For $z = 1$, $|z| = |1| = 1 < 2$. Hence, $z = 1$ lies inside C.

For $z = -\frac{3}{2}$, $\left|-\frac{3}{2}\right| = \frac{3}{2} < 2$. Hence, $z = -\frac{3}{2}$ lies inside C.

$z = 1$ is a pole of order 2.

$$\therefore \text{Res}\{f(z);1\} = \frac{1}{(2-1)!}\lim_{z\to 1}\frac{d}{dz}\Big[(z-1)^2 f(z)\Big]$$

$$= \lim_{z\to 1}\frac{d}{dz}\left[\frac{12z-7}{2z+3}\right]$$

$$= \lim_{z\to 1}\left[\frac{(2z+3)(12)-(12z-7)(2)}{(2z+3)^2}\right]$$

$$= \lim_{z\to 1}\left[\frac{24z+36-24z+14}{(2z+3)^2}\right]$$

$$= \lim_{z\to 1}\left[\frac{50}{(2z+3)^2}\right]$$

$$= \frac{50}{25} = 2$$

$z = -\frac{3}{2}$ is a simple pole.

$$\begin{aligned}
\text{Res}\left\{f(z); \frac{-3}{2}\right\} &= \lim_{z\to\frac{-3}{2}} \left(z+\frac{3}{2}\right) f(z) \\
&= \lim_{z\to\frac{-3}{2}} \left(z+\frac{3}{2}\right) \frac{12z-7}{(z-1)^2(2z+3)} \\
&= \lim_{z\to\frac{-3}{2}} \frac{12z-7}{2(z-1)^2} \\
&= \frac{12(\frac{-3}{2})-7}{2(\frac{-3}{2}-1)^2} \\
&= \frac{-25}{25/2} = -2.
\end{aligned}$$

Hence by Cauchy's residue theorem,

$$\oint_C f(z)dz = 2\pi i \times \big(\text{sum of the residues at the poles within } C\big)$$

$$\Rightarrow \oint_C \frac{12z-7}{(2z+3)(z-1)^2} dz = 2\pi i\big[2-2\big] = 0.$$

3. b. Evaluate $\int_0^\infty \frac{1}{(x^2+a^2)^2} dx$.

Solution: To evaluate the given integral, consider

$$\oint_C \frac{dz}{(z^2+a^2)^2} dz = \oint_C f(z)dz$$

where C is the contour consisting of the upper semi-circle C_R of radius R together with the part of the real axis from $-R$ to R.

$$\therefore \oint_C f(z)dz = \int_{C_R} f(z)dz + \int_{-R}^{R} f(x)dx. \quad \cdots\cdots \quad (1)$$

The poles of $f(z)$ are given by $(z^2+a^2)^2 = 0$

$$\Rightarrow z^2 + a^2 = 0 \Rightarrow z = \pm ai \text{ (twice)}$$

which are poles of order 2.

$z = -\frac{3}{2}$ is a simple pole.

$$\begin{aligned}
\text{Res}\left\{f(z); \frac{-3}{2}\right\} &= \lim_{z \to \frac{-3}{2}} \left(z + \frac{3}{2}\right) f(z) \\
&= \lim_{z \to \frac{-3}{2}} \left(z + \frac{3}{2}\right) \frac{12z - 7}{(z-1)^2(2z+3)} \\
&= \lim_{z \to \frac{-3}{2}} \frac{12z - 7}{2(z-1)^2} \\
&= \frac{12(\frac{-3}{2}) - 7}{2(\frac{-3}{2} - 1)^2} \\
&= \frac{-25}{25/2} = -2.
\end{aligned}$$

Hence by Cauchy's residue theorem,

$$\oint_C f(z)dz = 2\pi i \times \big(\text{sum of the residues at the poles within } C\big)$$

$$\Rightarrow \oint_C \frac{12z - 7}{(2z+3)(z-1)^2} dz = 2\pi i\big[2 - 2\big] = 0.$$

3. b. Evaluate $\int_0^\infty \frac{1}{(x^2 + a^2)^2} dx$.

Solution: To evaluate the given integral, consider

$$\oint_C \frac{dz}{(z^2 + a^2)^2} dz = \oint_C f(z)dz$$

where C is the contour consisting of the upper semi-circle C_R of radius R together with the part of the real axis from $-R$ to R.

$$\therefore \oint_C f(z)dz = \int_{C_R} f(z)dz + \int_{-R}^{R} f(x)dx. \qquad \cdots\cdots \quad (1)$$

The poles of $f(z)$ are given by $(z^2 + a^2)^2 = 0$

$$\Rightarrow z^2 + a^2 = 0 \Rightarrow z = \pm ai \text{ (twice)}$$

which are poles of order 2.

But only $z = ai$ lies inside C.

$$\therefore \text{Res}\{f(z); ia\} = \frac{1}{(2-1)!} \lim_{z \to ia} \frac{d}{dz}\{(z-ia)^2 f(z)\}$$
$$= \lim_{z \to ia} \frac{d}{dz}\left\{(z-ia)^2 \frac{1}{(z^2+a^2)^2}\right\}$$
$$= \lim_{z \to ia} \frac{d}{dz}\left\{\frac{1}{(z+ia)^2}\right\}$$
$$= \lim_{z \to ia}\left\{-\frac{2}{(z+ia)^3}\right\}$$
$$= -\frac{2}{(ia+ia)^3}$$
$$= -\frac{2}{(2ai)^3} = -\frac{2}{8a^3 i^3} = \frac{1}{4a^3 i}$$

Hence by Cauchy's residue theorem, we have

$$\oint_C f(z)dz = 2\pi i \times \big(\text{sum of the residues of } f(z) \text{ at poles within } C\big)$$
$$= 2\pi i \left(\frac{1}{4a^3 i}\right)$$
$$= \frac{\pi}{2a^3}.$$

Substituting in (1), we get

$$\int_{-R}^{R} f(x)dx + \int_{C_R} f(z)dz = \frac{\pi}{2a^3}.$$

But $\int_{C_R} f(z)dz \to 0$ as $z = Re^{i\theta}$ and $R \to \infty$.

Hence

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{1}{(x^2+a^2)^2}dx = \frac{\pi}{2a^3}$$
$$\Rightarrow \int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{2a^3}$$
$$\Rightarrow 2\int_{0}^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{2a^3}$$
$$\Rightarrow \int_{0}^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}$$

4. a. Expand $f(z) = \dfrac{z^2-1}{(z+2)(z+3)}$ in the region $2 < |z| < 3$.

Solution: Let

$$f(z) = \frac{z^2-1}{(z+2)(z+3)} = 1 - \frac{5z+7}{(z+2)(z+3)}$$

Let $\dfrac{5z+7}{(z+2)(z+3)} = \dfrac{A}{z+2} + \dfrac{B}{z+3}$
$\Rightarrow A(z+3) + B(z+2) = 5z+7$

Putting $z=-2,\ A=-3 \Rightarrow A=-3$

Putting $z=-3,\ -B=-8 \Rightarrow B=8.$

$$\therefore f(z) = 1 - \left[\frac{-3}{z+2} + \frac{8}{z+3}\right] = 1 + \frac{3}{z+2} - \frac{8}{z+3}.$$

$f(z)$ is not analytic at $z=-2$ and $z=-3$.

$f(z)$ is analytic in the annular region $2<|z|<3$ about $z=0$.

When $2<|z|<3$. For $2<|z| \Rightarrow \dfrac{2}{|z|}<1$ and $|z|<3 \Rightarrow \left|\dfrac{z}{3}\right|<1$. Hence

$$\begin{aligned} f(z) &= 1 + \frac{3}{z\left(1+\frac{2}{z}\right)} - \frac{8}{3\left(1+\frac{z}{3}\right)} \\ &= 1 + \frac{3}{z}\left(1+\frac{2}{z}\right)^{-1} - \frac{8}{3}\left(1+\frac{z}{3}\right)^{-1} \\ &= 1 + \frac{3}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n - \frac{8}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{3}\right)^n. \end{aligned}$$

4. b. Find the sum of residues of $f(z) = \dfrac{z^3}{(z-2)^2(z-3)}$ at its poles.

Solution: The poles of $f(z)$ are given by $(z-2)^2(z-3)=0 \Rightarrow z=2, z=3.$

$z=2$ is a pole of order 2 and $z=3$ is a simple pole.

Residue of $f(z)$ at $z=3$ is

$$\text{Res}\{f(z);3\} = \lim_{z\to 3}(z-3)f(z) = \lim_{z\to 3}\frac{z^3}{(z-2)^2} = 27.$$

Also residue of $f(z)$ at $z = 2$ is

$$\begin{aligned}
\text{Res}\{f(z); 2\} &= \lim_{z\to 2}\frac{d}{dz}\{(z-2)^2 f(z)\} \\
&= \lim_{z\to 2}\frac{d}{dz}\left(\frac{z^3}{z-3}\right) \\
&= \lim_{z\to 2}\frac{(z-3)(3z^2) - z^3\cdot 1}{(z-3)^2} \\
&= \lim_{z\to 2}\frac{2z^3 - 9z^2}{(z-2)^2} \\
&= -20.
\end{aligned}$$

Sum of the residues$=27 - 20 = 7$.

UNIT-III

5. a. In a certain college, 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body. What is the probability that mathematics is being studied? If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl?

Solution: Define the events:

E_1: The student is a girl

E_1: The student is a boy

M: The student is studied mathematics.

$$\therefore P(E_1) = 60\% = 0.6 \text{ and } P(E_2) = 40\% = 0.4$$

Probability that mathematics is studied given that the student is a boy

$$P(M|E_2) = 25\% = \frac{25}{100} = 0.25$$

Similarly,

$$P(M|E_1) = 10\% = \frac{10}{100} = 0.1.$$

(i) By theorem on total probability, probability that mathematics is studied

$$\begin{aligned}
P(M) &= P(E_1)P(M|E_1) + P(E_2)P(M|E_2) \\
&= (0.6)(0.1) + (0.4)(0.25) = 0.16
\end{aligned}$$

(ii) By Baye's theorem, probability that a mathematics student is a girl

$$\begin{aligned} P(E_1|M) &= \frac{P(E_1)P(M|E_1)}{P(M)} \\ &= \frac{0.6 \times 0.1}{0.16} \\ &= 0.375. \end{aligned}$$

5. b. A random variable X is defined as sum of the two numbers on the faces when two dice are thrown. Find the mean of X.

Solution: When two dice are thrown, total number of outcomes is $6 \times 6 = 36$. In this case, sample space

$$S = \left\{ \begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix} \right\}$$

If the random variable X assigns the sum of its numbers in S.

$$X(s) = X(a,b) = a + b = \{2,3,4,5,6,7,8,9,10,11,12\}.$$

For sum 2, the favorable case is (1, 1), so

$$p(2) = P(X = 2) = \frac{1}{36}$$

For sum 3, the favorable cases are (1, 2), (2, 1), so

$$p(3) = P(X = 3) = \frac{2}{36}$$

For sum 4, the favorable cases are (1, 3), (3, 1), (2, 2), so

$$p(4) = P(X = 4) = \frac{3}{36}$$

For sum 5, the favorable cases are (1, 4), (4, 1), (2, 3), (3, 2), so

$$p(5) = P(X = 5) = \frac{4}{36}$$

For sum 6, the favorable cases are (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), so

$$p(6) = P(X = 6) = \frac{5}{36}$$

For sum 7, the favorable cases are (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), so

$$p(7) = P(X = 7) = \frac{6}{36}$$

For sum 8, the favorable cases are (2, 6), (6, 2), (3, 5), (5, 3), (4, 4), so

$$p(8) = P(X = 8) = \frac{5}{36}$$

For sum 9, the favorable cases are (3, 6), (6, 3), (4, 5), (5, 4), so

$$p(9) = P(X = 9) = \frac{4}{36}$$

For sum 10, the favorable cases are (4, 6), (6, 4), (5, 5), so

$$p(10) = P(X = 10) = \frac{3}{36}$$

For sum 11, the favorable cases are (5, 6), (6, 5), so

$$p(11) = P(X = 11) = \frac{2}{36}$$

For sum 12, the favorable case is (6, 6), so

$$p(12) = P(X = 12) = \frac{1}{36}$$

$\therefore$ The discrete probability distribution is

$X = x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(X = x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\therefore$ Mean $\mu = E(X) = \sum_i x_i p_i$

$$\begin{aligned} &= 2(1/36) + 3(2/36) + 4(3/36) + 5(4/36) + 6(5/36) + 7(6/36) \\ &\quad + 8(5/36) + 9(4/36) + 10(3/36) + 11(2/36) + 12(1/36) \\ &= \frac{1}{36}[2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12] \\ &= \frac{252}{36} \\ &= 7. \end{aligned}$$

6. a. The probability of a man hitting a target is 1/3.

(i) If he fires 5 times, what is the probability of his hitting the target at least twice?

(ii) How many times must he fire so that the probability of his hitting the target at least once is greater than 90%?

Solution: The probability of a man hitting a target $p = \frac{1}{3}$

The probability of a man not hitting a target $q = 1 - p = \frac{2}{3}$

Number of trails= $n = 5$

Here the distribution is of successful hit or failure. Hence it is binomial distribution. Probability of hitting the target x times out of 5 times

$$P(X = x) = \binom{n}{x} p^x q^{n-x} = \binom{5}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}, \quad x = 0, 1, \cdots, 5$$

(i) If the man fires 5 times, probability of his hitting the target at least twice is given by

$$\begin{aligned} P(X \geq 2) &= 1 - P(X < 2) \\ &= 1 - \left[P(X = 0) + P(X = 1)\right] \\ &= 1 - \binom{5}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^5 - \binom{5}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^4 \\ &= 1 - \frac{32}{243} - 5 \times \frac{1}{3} \times \frac{16}{81} \\ &= 1 - \frac{32}{243} - \frac{80}{243} \\ &= \frac{131}{243} \\ &= 0.5391. \end{aligned}$$

(ii) The number of times he should he fire, so that the probability of hitting the target at least once is greater than 90% is calculated as follows:

$$P(X \geq 1) > 90\%$$
$$\Rightarrow P(X \geq 1) > 0.9$$
$$\Rightarrow 1 - P(X < 1) > 0.9$$
$$\Rightarrow 1 - P(X = 0) > 0.9$$
$$\Rightarrow P(X = 0) < 0.1$$
$$\Rightarrow \binom{n}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n < 0.1$$
$$\Rightarrow \left(\frac{2}{3}\right)^n < 0.1$$

For $n = 6$, $\left(\frac{2}{3}\right)^6 = 0.088 < 0.1$.

Hence, the man must fire 6 times so that the probability of hitting the target at least once is more than 90%.

6. b. If the masses of 300 students are normally distributed with a mean of 68 kg and standard deviation of 3 kg, how many students have masses

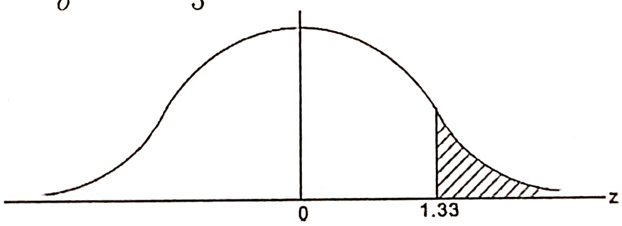
(i) greater than 72 kg (ii) less than or equal to 64 kg (iii) between 65 and 71 kg inclusive?

Solution: Let μ and σ be the mean and standard deviation of the distribution.

Then $\mu = 68$ and $\sigma = 3$.

Let X be the random variable which denotes the masses of a student.

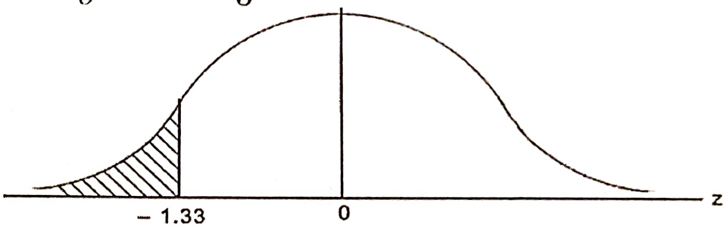
(i) When $X = 72$, $Z = \frac{X - \mu}{\sigma} = \frac{72 - 68}{3} = 1.33$.



$$\therefore P(X > 72) = P(Z > 1.33)$$
$$= 0.5 - P(0 \leq Z \leq 1.33)$$
$$= 0.5 - 0.4082$$
$$= 0.0918.$$

$\therefore$ Number of students with masses more than 72 kg$=NP(X > 72) = 300 \times 0.0918 = 27.54 \cong 28$ (approximately).

(ii) When $X = 64$, $Z = \dfrac{X-\mu}{\sigma} = \dfrac{64-68}{3} = -1.33$.

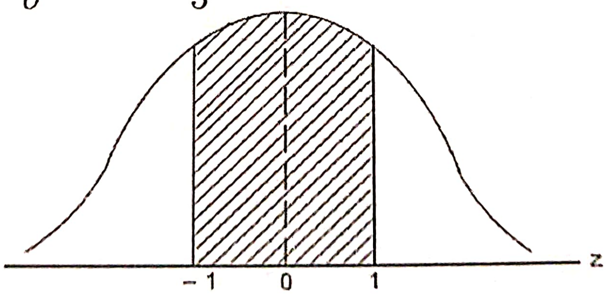


$$\begin{aligned}\therefore P(X \leq 64) &= P(Z \leq -1.33)\\ &= 0.5 - P(-1.33 \leq Z \leq 0)\\ &= 0.5 - P(0 \leq Z \leq 1.33)\\ &= 0.5 - 0.4082\\ &= 0.0918.\end{aligned}$$

$\therefore$ Number of students with masses less than or equal to 64 kg$=NP(X \leq 64) = 300 \times 0.0918 = 27.54 \cong 28$ (approximately).

(iii) When $X = 65$, $Z = \dfrac{X-\mu}{\sigma} = \dfrac{65-68}{3} = -1$

When $X = 71$, $Z = \dfrac{X-\mu}{\sigma} = \dfrac{71-68}{3} = 1$.



$$\begin{aligned}\therefore P(65 \leq X \leq 71) &= P(-1 \leq Z \leq 1)\\ &= P(-1 \leq Z \leq 0) + P(0 \leq Z \leq 1)\\ &= P(0 \leq Z \leq 1) + P(0 \leq Z \leq 1)\\ &= 2P(0 \leq Z \leq 1)\\ &= 2(0.3413)\\ &= 0.6826.\end{aligned}$$

$\therefore$ Number of students with masses between 65 kg 71 kg$=NP(65 \leq X \leq 71) =$

$300 \times 0.6826 = 204.78 \cong 205$. (approximately).

UNIT-IV

7. a. A population consists of five numbers 2, 3, 6, 8 and 11. Consider all possible samples of size two which can be drawn without replacement from this population. Find

(i) The mean of the population.

(ii) The standard deviation of the population.

(iii) The mean of the sampling distribution of means

(iv) The standard deviation of sampling distribution of means.

Solution: Given population 2, 3, 6, 8 and 11.

(i). Mean of the population,

$$\mu = \frac{2+3+6+8+11}{5} = \frac{30}{5} = 6.$$

(ii). Variance of the population,

$$\begin{aligned}\sigma^2 &= \sum_{i=1}^{5} \frac{(x_i-\mu)^2}{N} \\ &= \frac{(2-6)^2+(3-6)^2+(6-6)^2+(8-6)^2+(11-6)^2}{5} \\ &= \frac{16+9+0+4+25}{5} = 10.8\end{aligned}$$

$$\therefore \sigma = \sqrt{10.8} = 3.29.$$

Sampling without replacement (finite population)

Population size $N = 5$ and sample size $n = 2$.

$\therefore {}^N C_n = {}^5 C_2 = 10$ samples can be drawn from the population without replacement.

All possible samples of size two that can be drawn without replacement are

$$\left\{\begin{matrix} (2,3) & (2,6) & (2,8) & (2,11) \\ & (3,6) & (3,8) & (3,11) \\ & & (6,8) & (6,11) \\ & & & (8,11) \end{matrix}\right\}$$

The corresponding sample means are

$$\left\{\begin{array}{cccc} 2.5 & 4 & 5 & 6.5 \\ & 4.5 & 5.5 & 7 \\ & & 7 & 8.5 \\ & & & 9.5 \end{array}\right\} \quad \cdots\cdots \quad (1)$$

(iii). The mean of the sampling distribution of means

$$\begin{aligned} \mu_{\overline{X}} &= \frac{\text{Sum of all sample means in (1)}}{10} \\ &= \frac{2.5+4+5+6.5+4.5+5.5+7+7+8.5+9.5}{10} = \frac{60}{10} = 6. \end{aligned}$$

$\therefore$ Mean of the population=Mean of the sampling distribution of means i.e., $\mu_{\overline{X}} = \mu$.

(iv) The variance of sampling distribution of means

$$\sigma^2_{\overline{X}} = \frac{(2.5-6)^2 + (4-6)^2 + \cdots + (9.5-6)^2}{10} = \frac{40.5}{10} = 4.05$$

$\therefore$ Standard deviation $\sigma_{\overline{X}} = \sqrt{4.05} = 2.01.$

$$\begin{aligned} \text{S.E.} &= \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} \\ &= \frac{3.29}{\sqrt{2}}\sqrt{\frac{5-2}{5-1}} = 2.01 \end{aligned}$$

$\therefore$ Standard deviation of sampling distribution of means=Standard Error of Means.

7. b. Determine the probability that the sample mean area covered by sample of 40 one litre paint boxes will be between 510 to 520 square feet given that a one litre of such paint box covers on the average 513.3 square feet with s.d. of 31.5 s.ft.

Solution: Given that

n=Size of the sample=40

μ=Mean of the population=513.3

σ=S.D of the population=31.5.

We now find $P(510 \leq \overline{x} \leq 520)$.

We know that $z = \dfrac{\overline{x}-\mu}{\sigma/\sqrt{n}}$.

If $\overline{x}_1 = 510$, $z_1 = \dfrac{\overline{x}_1-\mu}{\sigma/\sqrt{n}} = \dfrac{510-513.3}{31.5/\sqrt{40}} = -0.6$

If $\overline{x}_2 = 520$, $z_2 = \dfrac{\overline{x}_2 - \mu}{\sigma/\sqrt{n}} = \dfrac{520 - 513.3}{31.5/\sqrt{40}} = 1.4.$

$$\begin{aligned}
\therefore P(510 \leq \overline{x} \leq 520) &= P(z_1 \leq z \leq z_2) \\
&= P(-0.6 \leq z \leq 1.4) \\
&= P(-0.6 \leq z \leq 0) + P(0 \leq z \leq 1.4) \\
&= P(0 \leq z \leq 0.6) + P(0 \leq z \leq 1.4) \\
&= 0.2258 + 0.4192 \\
&= 0.645
\end{aligned}$$

8. a. A random sample of size 100 is taken from a population with $\sigma = 5.1$. Given that the sample mean is $\overline{x} = 21.6$. Construct a 95% confidence interval for the population mean μ.

Solution: Given that σ=Standard deviation=5.1

$\overline{x}$=Sample mean=21.6

n=Sample size=100.

Confidence limit=95%.

$Z_{\alpha/2} = 1.96$ (for 95%)

Let μ be the mean of the population.

$\therefore$ The confidence interval is given by

$$\overline{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

Now $\overline{x} - Z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} = 21.6 - \dfrac{1.96 \times 5.1}{\sqrt{100}} = 20.6$

and $\overline{x} + Z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} = 21.6 + \dfrac{1.96 \times 5.1}{\sqrt{100}} = 22.6$

Hence $(20.6, 22.6)$ is the confidence interval for the population mean μ.

8. b. A random sample of size 100 has a standard deviation is 5. What can you say about the maximum error with 95% confidence?

Solution: The maximum error $E = Z_{\alpha/2}\left(\dfrac{\sigma}{\sqrt{n}}\right)$.

Given that $n = 100$, S.D $\sigma = 5$.

Confidence limit = 95%

$\Rightarrow (1 - \alpha)100 = 95$

$\Rightarrow 1 - \alpha = 0.95$

$\Rightarrow \alpha = 1 - 0.95 = 0.05$

$\Rightarrow \alpha/2 = 0.025$

$\Rightarrow Z_{\alpha/2} = 1.96.$

$$\therefore \text{ Maximum error } E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = (1.96)\left(\frac{5}{10}\right) = 0.98.$$

UNIT-V

9. a. A sample of heights of 6400 English men has a mean of 67.85 inches and standard deviation 2.56 inches, while a sample of heights of 1600 Australians has a mean of 68.55 inches and standard deviation 2.52 inches. Do the data indicate that Australians are on the average taller than the English men?

Solution: We are given:

Size of the first sample $n_1 = 6400$

Size of the second sample $n_2 = 1600$

Mean of the first sample $\overline{x}_1 = 67.85$

Mean of the second sample $\overline{x}_2 = 68.55$

Standard deviation of the first sample $s_1 = 2.56$

Standard deviation of the second sample $s_2 = 2.52$

1. Null hypothesis H_0: $\mu_1 = \mu_2$, i.e., there is no significant difference in heights of Australians and English men
2. Alternative hypothesis H_1: $\mu_1 < \mu_2$ (Left-tailed test)
3. Level of significance: $\alpha = 0.05$

4. Test statistic:

$$
\begin{aligned}
Z &= \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\
&= \frac{67.85 - 68.55}{\sqrt{\frac{(2.56)^2}{6400} + \frac{(2.52)^2}{1600}}} \\
&= \frac{-0.7}{\sqrt{\frac{6.5536}{6400} + \frac{6.35}{1600}}} \\
&= \frac{-0.7}{\sqrt{0.001 + 0.004}} \\
&= -9.9.
\end{aligned}
$$

i.e., calculated value of $|Z| = 9.9$.

5. Critical value: Since $\alpha = 0.05 \Rightarrow Z_\alpha = 1.645$ (for left-tailed test).

6. Decision: Since calculated value of Z is greater than tabulated value of Z at 5% level of significance, the null hypothesis H_0 has to be rejected.

Conclusion: Australians are on the average taller than English men.

9. b. The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches?

Solution: Given $n = 10, \mu = 64$.

Calculation for sample means and S

x	$x-\overline{x}$	$(x-\overline{x})^2$
70	4	16
67	1	1
62	-4	16
68	2	4
61	-5	25
68	2	4
70	4	16
64	-2	4
64	-2	4
66	0	0
$\sum x = 660$		$\sum(x-\overline{x})^2 = 90$

$$\text{Sample mean } \overline{x} = \frac{\sum x}{n} = \frac{660}{10} = 66.$$

We know that

$$S^2 = \frac{1}{n-1}\sum(x-\overline{x})^2 = \frac{1}{9} \times 90 = 10 \Rightarrow S = \sqrt{10} = 3.16.$$

1. Null hypothesis H_0: The average height is not greater than 64 inches

   i.e., $\mu = 64$ inches

2. Alternative hypothesis H_1: $\mu > 64$ (Right-tailed test)
3. Level of significance: $\alpha = 0.05$
4. Test statistic:

$$\begin{aligned} t &= \frac{\overline{x}-\mu}{S/\sqrt{n}} \\ &= \frac{66-64}{3.16/\sqrt{10}} \\ &= \frac{2 \times \sqrt{10}}{3.16} \\ &= 2. \end{aligned}$$

i.e., calculated value of $|t| = 2$.

5. Critical value: Degrees of freedom $\nu = n - 1 = 10 - 1 = 9$.

The table value of t for 9 degrees of freedom at 5% level of significance for right-tailed test is 1.833.

6. Decision: Since calculated value of $t >$tabulated value of t, the null hypothesis H_0 has to be rejected.

Conclusion: The average height is greater than 64 inches.

10. a. In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

Solution: Here $n = 600$.

X=Number of smokers in city=325

p=Sample proportion of smokers in city=$\frac{X}{n} = \frac{325}{600} = 0.5417$

P=Population proportion of smokers in city=$\frac{1}{2} = 0.5$

$Q = 1 - P = 0.5$

1. Null hypothesis H_0: The number of smokers and non-smokers are equal in the city i.e., $P = 0.5$
2. Alternative hypothesis H_1: $P > 0.5$ (Right-tailed test)
3. Level of significance: $\alpha = 0.05$
4. Test statistic:

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.5417 - 0.5}{\sqrt{\frac{0.5\times0.5}{600}}} = 2.04.$$

i.e., calculated value of $|Z| = 2.04$.

5. Critical value: Since $\alpha = 0.05 \Rightarrow Z_\alpha = 1.645$ (for right-tailed test).

6. Decision: Since calculated value of Z is greater than tabulated value of Z at 5% level of significance, the null hypothesis H_0 has to be rejected.

Conclusion: Proportion of smokers in city is more than 50% and majority of men in the city are smokers.

10. b. The following random samples are measurements of the heat producing capacity (in millions of calories per ton)of specimens of coal from two mines

Mine 1:	8260	8130	8350	8070	8340	
Mine 2:	7950	7890	7900	8140	7920	7840

Use 0.02 LOS to test whether it is reasonable to assume that the variances of the two population samples are equal.

Solution: Given $n_1 = 5$ and $n_2 = 6$.

Now $\overline{x} = \frac{\sum x_i}{n_1} = \frac{41150}{5} = 8230$ and $\overline{y} = \frac{\sum y_i}{n_2} = \frac{47640}{6} = 7940$.

x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$	y_i	$y_i - \overline{y}$	$(y_i - \overline{y})^2$
8260	30	900	7950	10	100
8130	−100	10000	7890	−50	2500
8350	120	14400	7900	−40	1600
8070	−160	25600	8140	200	40000
8340	110	12100	7920	−20	400
			7840	−100	10000
41150		63000	47640		54600

Here $\sum(x_i - \overline{x})^2 = 63000$ and $\sum(y_i - \overline{y})^2 = 54600$.

$$\therefore S_1^2 = \frac{\sum(x_i - \overline{x})^2}{n_1 - 1} = \frac{63000}{4} = 15750 \text{ and } S_2^2 = \frac{\sum(y_i - \overline{y})^2}{n_2 - 1} = \frac{54600}{5} = 10920.$$

***F*-test:**

1. Null hypothesis H_0: $\sigma_1^2 = \sigma_2^2$, i.e., the variances of success are equal
2. Alternative hypothesis H_1: $\sigma_1^2 \neq \sigma_2^2$ (two-tailed test)
3. Level of significance: $\alpha = 0.05$
4. Test statistic:

$$F = \frac{S_1^2}{S_2^2} = \frac{15750}{10925} = 1.44 \qquad [\because S_1^2 > S_2^2]$$

Therefore, calculated value of F=1.44

5. Critical value: Degrees of freedom = $(n_1 - 1, n_2 - 1) = (4, 5)$.

Tabulated value of F for $(4, 5)$ d.f at 5% level of significance is 5.19

6. Decision: Since calculated value of F is less than tabulated value of F, we accept the null hypothesis H_0.

Conclusion: The variances of two samples are equal.

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